PROBABILITY

Consider the following actions

1. A ball is thrown in the air
2. A spoonful sugar is put in hot water
3. A coin is tossed
4. Three friends play a game

Out of 4 above actions, results of action1 & 2 is known certainly before the action where as result of action 3 & 4 can not be told with certainty unless the action takes place.

We can say outcome of a toss can be either Head or Tail . There are some chances that the outcome is Head and some chances that it is Tail.

Similarly when 3 friends play a game there is an element of chance that each one will win the game.

This element of chance is known as Probability in Statistics.

Basic Concepts

Statistical Trial

An experiment when repeated under identical conditions does not give unique result but may result into one of the several possible outcomes , it is known as Statistical Trial

Event

Outcomes of such an experiment are known as events.

Rolling of a dice is an experiment , getting numbers 1, 2, 3,4,5,6 on the uppermost face is an event.

Drawing a card from a well shuffled pack of cards is an experiment , getting a King of Club is an Event

Sample Space

Set of all possible outcomes of an experiment is called Sample Space for that experiment.

It is usually denoted by S.

Elements of Sample Space are called as sample points

Eg In an experiment of tossing of 2 coins

S = { HH , HT , TH , TT}

In an experiment of rolling of a dice

S = { 1 , 2 , 3 , 4 , 5, 6 }

Any subset of sample Space is an Event

Eg In an experiment of tossing of 2 coins

S = { HH , HT , TH , TT}

A : Outcome of both the tosses are not identical

A = { HT , TH }

A is a subset of S , A is an Event

In an experiment of rolling of a dice

S = { 1 , 2 , 3 , 4 , 5, 6 }

A: Outcome is multiple of 3

A ={ 3 , 6}

A is a subset of S , A is an Event

Impossible Event

An event which can not occur under any circumstances is known as Impossible Event.

In an experiment of rolling of a dice ,

P : Outcome is 10

We know that this can not happen under any circumstances as numbers on a dice are ranging from 1 to 6 only .

Hence P is an Impossible Event

Certain Event

An event which is sure to occur under any circumstances is known as Certain Event.

In same experiment of rolling of a dice ,

Q: Outcome is < 7

We know that this will always happen under any circumstances as numbers on a dice are ranging from 1 to 6 only .

Hence Q is Certain Event

Mutually Exclusive Events

If events A & B are such that they can not occur simultaneously or they have no sample point common, events A & B are mutually Exclusive Events.

In an experiment of rolling of a dice

S = { 1 , 2 , 3 , 4 , 5, 6 }

A: Outcome is an odd number

A ={1 , 3 , 5}

B: Outcome is an even number

B = { 2, 4. 6 }

$A \cap B =$ Null Set

A & B are Mutually Exclusive Events

Equally Likely Events

If events A & B are such that they have equal chances of occurrence then events A & B are said to be Equally Likely Events

Eg In an experiment of tossing of 2 coins

S = { HH , HT , TH , TT}

A : Outcome of both the tosses are not identical

A = { HT , TH }

B : Outcome of both the tosses are identical

B = { HH , TT }

Exhaustive Events

If events A & B are such that sample points of both the events together constitute Sample space then events A & B are said to be Exhaustive Events

In an experiment of rolling of a dice

S = { 1 , 2 , 3 , 4 , 5, 6 }

A: Outcome is < 4

A ={1 , 2 , 3}

B: Outcome is >= 3

B = { 3, 4 , 5 , 6 }

A U B = { 1 , 2 , 3, , 4 , 5 , 6 } = S

Classical definition of Probability

If an experiment can result into any one of n mutually exclusive , equally likely , exhaustive outcomes of which m are favourable to an event A , then the probability of occurrence of event A is $\frac{m}{n}$ and is denoted as P(A)

P(A) = $\frac{m}{n}$

We have 0 <= m <= n

$$\frac{0}{n} <= \frac{m}{n} <= \frac{n}{n}$$

$$0 <= \frac{m}{n} <= 1$$

Probability lies between 0 and 1

Coin and dice problems

Q1. Two coins are tossed simultaneously. Find the probability of occurrence of

i. 2 heads
ii. 1 head

S = { HH , TH , HT , TT }

n(S) = 4

A : Heads

$A = \{HH\}$

$n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$

B : 1 Head

$B = \{HT , TH\}$

$n(B) = 2$

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$

Q.2 Three coins are tossed simultaneously. Find the probability of occurrence of

i. At least 2 heads
ii. No head

$S = \{ HHH , THH , HTH , HHT , HTT , THT , TTH, TTT \}$

$n(S) = 8$

A : At least 2 Heads

$A = \{HHH , THH , HTH , HHT \}$

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

B : No Head

$B = \{TTT \}$

$n(B) = 1$

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$

Q.3 Four coins are rolled simultaneously. Find the probability of occurrence of

i. 3 heads
ii. 2 Heads

To write S
4 Heads – HHHH
3 heads – THHH , HTHH , HHTH , HHHT
2 Heads – HHTT , THHT , TTHH , HTTH , THTH , HTHT
1 Head – HTTT , THTT , TTHT , TTTH
0 Head - TTTT
S = { HHHH , THHH , HTHH , HHTH , HHHT, HHTT , THHT , TTHH, HTTH , THTH , HTHT, HTTT , THTT , TTHT , TTTH, TTTT }
n(S) = 16
A : 3Heads
A = {THHH , HTHH , HHTH , HHHT}
n(A) = 4
$P(A) = \frac{n(A)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

B : 3Heads
B = {HHTT , THHT , TTHH , HTTH , HTHT , THTH}
n(B) = 6
$P(B) = \frac{n(B)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

Q.4 Two dice are rolled simultaneously , Find the probability that

i. Sum of numbers on uppermost face is 7
ii. Sum is 7 or 11
iii. Sum is <=5
iv. Sum is a perfect square

Sample Space

S = { (1,1) , (1,2) , (1,3) , (1,4) ,(1,5) ,(1,6),
(2,1) , (2,2) , (2,3) , (2,4) ,(2,5) ,(2,6),
(3,1) , (3,2) , (3,3) , (3,4) ,(3,5) ,(3,6),
(4,1) , (4,2) , (4,3) , (4,4) ,(4,5) ,(4,6),
(5,1) , (5,2) , (5,3) , (5,4) ,(5,5) ,(5,6),
(6,1) , (6,2) , (6,3) , (6,4) ,(6,5) ,(6,6) }

n(S) = 36

A : Sum of outcomes is 7

A = {(1,6) , (2,5), (3,4), (4,3) ,(5,2) ,(6,1)}

n(A) = 6

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

B : Sum of outcomes is 7 or 11

B= {(1,6) , (2,5), (3,4), (4,3) ,(5,2) ,(6,1), (5,6),(6,5)}

n(B) = 8

$P(B) = \frac{n(B)}{n(S)} = \frac{8}{36} = \frac{2}{9}$

C : Sum of outcomes is <=5

C= {(1,1) , (1,2), (2,1), (1,3) ,(2,2) ,(3,1), (1,4) ,(2,3), (3,2) , (4,1)}

n(C) = 10

$P(C) = \frac{n(C)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

D : Sum of outcomes is a perfect square

D : Sum of outcomes is 4,9

D = {(1,3) , (2,2), (3,1), (3,6) ,(4,5) ,(5,4), (6,3)}

n(D) = 7

$P(D) = \frac{n(D)}{n(S)} = \frac{7}{36}$

Permutation & Combination

Factorial

The product of first n natural numbers is called factorial n

n! = 1 x 2 x 3 x --------x (n-1) x n

n! = n (n-1) !

Permutations

Each different arrangements out of n elements taking r at a time is called permutation

$nPr = \frac{n!}{(n-r)!}$ (r<=n)

- **Combination**

Each of groups formed by selecting r elements out of n is called combination

$nCr = \frac{n!}{r!\,(n-r)!}$

A certain thing can be done in m ways , another thing can be done in n ways , both the things together can be done in m*n ways

Q1. If letters of word THURSDAY is to be arranged randomly , what is the probability that the arrangement

Begins with T

Begins with T and ends with U.

There are in all 8 letters in word THURSDAY

They can be arranged in 8! Ways

n(S) = 8!

A: Arrangement begins with T

Position of letter T is fixed. It can be done in only 1 way & remaining 7 letters can be arranged in 7! Ways

n(A) = 1 * 7!

$P(A) = \frac{n(A)}{n(S)} = \frac{7!}{8!} = \frac{(1*2*3*4*5*6*7)}{(1*2*3*4*5*6*7*8)} = \frac{1}{8}$

B: Arrangement begins with T and ends with U

Position of letter T & U is fixed. It can be done in only 1 way & remaining 6 letters can be arranged in 6! Ways

n(B) = 1 * 6!

$$P(B) = \frac{n(B)}{n(S)} = \frac{6!}{8!} = \frac{(1*2*3*4*5*6)}{(1*2*3*4*5*6*7*8)} = \frac{1}{7*8} = \frac{1}{56}$$

Q2. 6 persons including a couple are to be seated for a photograph in a row. Find a probability of photograph in which the couple is sitting together

There are in all 6 people

They can be arranged in 6! Ways

n(S) = 6!

A: The couple is sitting together

Two people in the couple can be arranged within themselves in 2! Ways .Consider couple as one unit remaining 4 people + 1 couple in all 5 units which can be arranged in 5! Ways

n(A) = 2! * 4!

$$P(A) = \frac{n(A)}{n(S)} = \frac{2! * 5!}{6!} = \frac{(1*2)(1*2*3*4*5)}{(1*2*3*4*5*6)} = \frac{1}{3}$$

Q 3.If letters of word FATHER is to be arranged randomly , what is the probability that the letters A & R are at the extremes.

There are in all 6 letters in word FATHER

They can be arranged in 6! Ways

n(S) = 6!

A: letters A & R are at the extremes

Letters A & R can be arranged within themselves in 2! Ways & remaining 4 letters can be arranged in 4! Ways

n(A) = 2! * 4!

$$P(A) = \frac{n(A)}{n(S)} = \frac{2! * 4!}{6!} = \frac{(1*2)(1*2*3*4)}{(1*2*3*4*5*6)} = \frac{1}{15}$$

Q 4.6 boys and 5 girls are to be seated in a row. Find the probability of an arrangement in which no two girls are sitting together .

- 6 boys and 5 girls together 11 people -can be arranged in 11! Ways

  n(S) = 11 !

A : No two girls are sitting together

6 boys can be arranged in 6! Ways.

• B • B • B • B • B • B •

5 girls can take any 5 of 7 positions shown by •

This can be done in 7P5 Ways.

n(A) = 6! * 7P5

$P(A) = \frac{n(A)}{n(S)} = \frac{6! * 7P5}{11!} = \frac{6! * 7!}{11! * 2\,!} \frac{(1*2*3*4*5*6)}{(1*2)(8*9*10*11)} = \frac{1}{22}$

Q 5.A staff of a department of a company consists of a manager , an officer & 10 clerks. A committee of 4 is to be selected from the department. Find the probability of a committee which includes

i. The manager
ii. The manager but not officer
iii. Neither manager nor officer

Ans: A committee of 4 is to be selected from 12. it can be done in $^{12}C_4$ ways.

$n(S) = {}^{12}C_4 = \frac{12!}{4! * 8!}$

A : A committee includes manager

There is only 1 manager , he can be selected in only 1 way. Remaining 3 people in the committee can be selected from 11 available in $^{11}C_3$ ways.

$n(A) = {}^{11}C_3 = \frac{11!}{3! * 8!}$

$P(A) = \frac{n(A)}{n(S)} = \frac{11!}{3! * 8!} \text{ X } \frac{4! * 8!}{12!} = \frac{4}{12} = \frac{1}{3}$

B : A committee includes manager but not officer.

There is only 1 manager , he can be selected in only 1 way. Remaining 3 people in the committee can be selected from 11 – 1 officer = 10 officer available in $^{10}C_3$ ways.

$n(B) = {}^{10}C_3 = \frac{10!}{3! * 7!}$

$P(B) = \frac{n(B)}{n(S)} = \frac{10!}{3! * 7!} \text{ X } \frac{4! * 8!}{12!} = \frac{4 * 8}{11 * 12} = \frac{8}{33}$

C : A committee can include neither manager nor officer.

There is 1 manager and 1 officer . We need to select all 4 members from remaining 10 people in $^{10}C_4$ ways.

$n(C) = {}^{10}C_4 = \frac{10!}{4! * 6!}$

$P(C) = \frac{n(C)}{n(S)} = \frac{10!}{4! * 6!} X \frac{4! * 8!}{12!} = \frac{7 * 8}{11 * 12} = \frac{14}{33}$

Q 6. A bag contains 8 red and 3 white marbles . If 3 marbles are drawn at random , what is the probability that

All three are Red

All three are White

2 Red and 1 white

3 marbles can be drawn from 11 in 11C3 ways

$n(S) = 11C3 = \frac{11!}{3! * 8!}$

A : All 3 are Red

$n(A) = 8C3 = \frac{8!}{3! * 5!}$

$P(A) = \frac{n(A)}{n(S)} = \frac{8!}{3! * 5!} X \frac{3! * 8!}{11!} = \frac{6 * 7 * 8}{9 * 10 * 11} = \frac{56}{165}$

B : All 3 are White

$n(B) = 3C3 = 1$

$P(B) = \frac{n(B)}{n(S)} = \frac{3! * 8!}{11!} = \frac{1 * 2 * 3}{9 * 10 * 11} = \frac{1}{165}$

C : 2Red and 1 White

$n(C) = {}^8C_2 * {}^3C_1$

$P(C) = \frac{n(C)}{n(S)} = \frac{8! * 3}{2! * 6} x \frac{8! * 3!}{11!} = \frac{7 * 8}{10 * 11} = \frac{28}{55}$

Complementary Events

- If two events A & B are such that they do not have anything in common and together they constitute Sample Space, then A & B are known as Complementary Events
- For Complementary Events , any Sample point from Sample Space has to belong to one and only one of A and B.
- Complementary Events are Mutually Exclusive as well as Exhaustive

- Complementary Events are denoted by A , $\bar{A}$
- If A and B are complementary events

  $P(A \cap B) = 0$, $P(A \cup B) = 1$

  $P(A) + P(B) = 1$

  If A and $\overline{A}$ are complementary then

  $P(A) + P(\overline{A}) = 1$

  $P(\overline{A}) = 1 - P(A)$

  $P(A) = 1 - P(\overline{A})$

  P(At least one) = 1 – P(None)

Laws of Probability

<u>Addition Theorem</u>

- If A & B are any two events associated with an experiment, then probability of occurrence of events A or B or both is given by
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A & B are mutually exclusive , $A \cap B$ is null set.
- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) + P(B)$

  For 3 events,

  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Conditional Probability

- Probability of event A given that B has already occurred is called conditional probability of event A given that event B has already occurred. It is denoted by P(A/B)
- conditional probability of event B given that event A has already occurred. It is denoted by P(B/A)

Multiplication Theorem

- If A & B are two events associated with an experiment , then the probability of simultaneous occurrence of event A & B is given by
- $P(A \cap B) = P(A) * P(B/A) = P(B) * P(A/B)$
- **Independent Events**

- If probability of event A is not affected by occurrence or non-occurrence of event B , then event A & B are called independent events
- If A & B are independent events ,
- P(A/B)= P(A) & P(B/A) = P(B)
- If A & B are independent events then
- $P(A \cap B) = P(A) * P(B)$

Q1. From past experience , it is known that A can solve 3 examples out of 5 , B can solve 4 examples out of 7. An example is given to both of them to solve independently.

Find the probability

i. The example remains unsolved
ii. The example is solved
iii. Only one of them solves the example

A: A Solves the example

$P(A) = \frac{3}{5}$, $P(\overline{A}) = \frac{2}{5}$

B: B Solves the example

$P(B) = \frac{4}{7}$, $P(\overline{B}) = \frac{3}{7}$

i. P(Problem is unsolved) = P(None)

$= P(\bar{A} \cap \bar{B})$

$= P(\overline{A}) * P(\overline{B})$

$= \frac{2}{5} * \frac{3}{7} = \frac{6}{35}$

ii. P(Problem is solved) = P (At least one solves)

= 1 – P (None)

$= 1 - \frac{6}{35} = \frac{29}{35}$

iii. P (Only 1 solves the example)

$= P(A \cap \overline{B}) + P(\bar{A} \cap B)$

$= P(A) * P(\overline{B}) + P(\bar{A}) * P(B)$

$= \frac{3}{5} * \frac{3}{7} + \frac{2}{5} * \frac{4}{7} = \frac{9}{35} + \frac{8}{35} = \frac{17}{35}$

Q2. A certain machine is made up of 3 independently working components A , B, C. The probability that these parts will go out of order at any point of time is 0.2 , 0.3 , 0.1 respectively.
What is the probability that machine will work satisfactorily if it is assumed that the machine works if

i. All 3 parts are working
ii. At least one part is working

- A: part A is working $P(\bar{A}) = 0.2$, $P(A) = 0.8$
- B: part B is working $P(\bar{B}) = 0.3$, $P(B) = 0.7$
- C: part C is working $P(\bar{C}) = 0.1$, $P(C) = 0.9$

i. $P(A \cap B \cap C) = P(A) * P(B) * P(C)$

$= 0.8 * 0.7 * 0.9$

$= 0.504$

ii. P(At least one)= 1 – P (None)

$= 1 - [P(\bar{A}) * P(\bar{B}) * P(\bar{C})]$

$= 1 - [0.2*0.3*0.1]$

$= 0.994$

Q 3. Three boys aim at a target. The probability that P hits the target is 0.3 , Q hits is 0.5 and R hits is 0.2. If all 3 hit the target independently , What is the probability that

i. All 3 hit the target
ii. At least one hits the target

P : P hits the target $P(P) = 0.3$, $P(\bar{P}) = 0.7$
Q : Q hits the target $P(Q) = 0.5$, $P(\bar{Q}) = 0.5$
R : R hits the target $P(R) = 0.2$, $P(\bar{R}) = 0.8$

- P(all 3 hit the target) = $P(P \cap Q \cap R)$

= P(P) * P(Q) * P(R)

= 0.3 * 0.5 * 0.2

= 0.3

P(At least one hits the target) = 1- P(None)

= 1 - P ($\bar{P} \cap \bar{Q} \cap \bar{R}$)

= 1 – [P($\bar{P}$) * P($\bar{Q}$) * P($\bar{R}$)]

= 1 – [0.7*0.5 * 0.8]

= 1 – 0.28

= 0.72

Odds in favour and against

- If odds in favour of A are a: b
  Then P(A) = $\frac{a}{a+b}$ $P(\bar{A}) = \frac{b}{a+b}$
- If odds against A are a: b
  Then P(A) = $\frac{b}{a+b}$ $P(\bar{A}) = \frac{a}{a+b}$
- If probability of occurrence of event A is p and probability of non-occurrence of event A is q , then odds in favour of A will be $\frac{p}{p+q}$: $\frac{q}{p+q}$

  p: q
- odds against A will be $\frac{q}{p+q}$: $\frac{p}{p+q}$ = q : p

Q 1. If odds are 3:2 against a person A living for next 20 years. The odds in favour of person B surviving for next 20 years is 3:4. What is the probability that at least one of A and B will survive for next 20 years?

- A: A living for next 20 years.
- **B**: B living for next 20 years.
- Odds against A are 3:2 so P(A) =$\frac{2}{5}$, P($\bar{A}$) = $\frac{3}{5}$
- Odds in favour of B are 3:4 so P(B) =$\frac{3}{7}$, P($\bar{B}$) = $\frac{4}{7}$
- P(At least one survives) = 1 – P (None)
  = 1 – P($\bar{A} \cap \bar{B}$)

$= 1 - P(\overline{A}) * P(\overline{B})$

$= 1 - [\frac{3}{5} * \frac{4}{7}] = \frac{23}{35}$

Q 2. If odds in favour of A hitting the target are 4:3 Odds against B hitting the target are 2:3. If Both of them shot at the target independently, Find the probability that the target is hit?

A: A hits the target

B: B hits the target

odds in favour of A hitting the target are 4:3

$P(A) = \frac{4}{7}$, $P(\bar{A}) = \frac{3}{7}$

Odds against B hitting the target are 2:3

$P(B) = \frac{3}{5}$, $P(\bar{B}) = \frac{2}{5}$

- P(target is hit) = 1 – (None)

$= 1 - P(\overline{A} \cap \overline{B})$

$= 1 - P(\overline{A}) * P(\overline{B})$

$= 1 - [\frac{3}{7} * \frac{2}{5}] = 1 - \frac{6}{35} = \frac{29}{35}$

Problems on Theorems of Probability

If $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{1}{2}$

Find P(A/B) , P(B/A)

We know that by Addition theorem we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{1}{2} = \frac{2}{5} + \frac{1}{3} - P(A \cap B)$

$P(A \cap B) = \frac{2}{5} + \frac{1}{3} - \frac{1}{2} = \frac{12+10-15}{30} = \frac{7}{30}$

By Multiplication theorem

$P(A \cap B) = P(A) * P(B/A)$

$\frac{7}{30} = \frac{2}{5} * P(B/A)$

$P(B/A) = \frac{7}{30} * \frac{5}{2} = \frac{7}{12}$

Similarly P(A∩B) = P(B) * P(A/B)

$\frac{7}{30} = \frac{1}{3}$ * P(A/B)

P(A/B) = $\frac{7}{30} * \frac{3}{1} = \frac{7}{10}$

Q2. If P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$, P(AUB) = $\frac{1}{6}$, Find P($\overline{A}$) and P(A∩ B)

- We know that by Addition theorem we have
  P(A U B) = P(A) + P(B) - P(A∩ B)
  $\frac{1}{6} = \frac{1}{2} + \frac{1}{3}$ - P(A∩ B)
  P(A∩ B) $= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{2}{3}$
  P($\overline{\text{A}}$) = 1- P(A) = 1 - $\frac{1}{2} = \frac{1}{2}$

Q3. If P(A) = $\frac{1}{4}$, P($\bar{B}$) = $\frac{4}{5}$, events A and b are independent , Find P(A U B) ,

P(A∩ B)

P($\overline{\text{B}}$) = 1- P(B)

P(B)= 1 - P($\overline{\text{B}}$)= 1- $\frac{4}{5} = \frac{1}{5}$

By Multiplication theorem since A & B are independent

P(A∩B) = P(A) * P(B)

$= \frac{1}{4} * \frac{1}{5} = \frac{1}{20}$

We know that by Addition theorem we have

P(A U B) = P(A) + P(B) - P(A∩ B)

$= \frac{1}{4} + \frac{1}{5} - \frac{1}{20} = \frac{5+4-1}{20} = \frac{8}{20} = \frac{2}{5}$

Discrete Random Variable

- If a random variable takes values x_1, x_2,x_3 ……., x_n with probabilities p_1, p_2, p_3 …..p_n then it is called discrete random variable.

Probability mass function –

If a discrete random variable X takes values $x_1, x_2, x_3 \ldots\ldots, x_n$ with probabilities $p_1, p_2, p_3 \ldots\ldots p_n$ then probability function P(x) is called probability mass function if $p_i >= 0$ & $\sum p_i = 1$

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- For probability mass function P(x)
  $P(x) >= 0$ for all x and $\sum p(x) = 1$

Expectation of random variable

If a discrete random variable X takes values $x_1, x_2, x_3 \ldots\ldots, x_n$ with probabilities $p_1, p_2, p_3 \ldots\ldots p_n$ then Expectation of random variable

$E(x) = x * p(x) = \sum x_i * p_i$

$E(x^2) = x^2 * p(x) = \sum x_i 2 * p_i$

Variance of random variable

If a discrete random variable X takes values $x_1, x_2, x_3 \ldots\ldots, x_n$ with probabilities $p_1, p_2, p_3 \ldots\ldots p_n$ then variance of random variable

$V(x) = E(X^2) - [E(X)]^2$

Q1. For the following distribution find

$P(X>2)$, $P(X<=1)$, $P(X = 2 \text{ or } 3)$, $E(X)$, $V(X)$

X	-2	-1	0	1	2	3
P(X)	0.1	0.2	0.2	0.3	0.15	0.05

- $P(x>2) = p(x = 3) = 0.05$
  $P(X <= 1) = P(x=-2) + P(X=-1) + P(X=0) + P(X=1)$
  $= 0.1+0.2+0.2+0.3 = 0.8$
  $P(X=2 \text{ OR } 3) = P(X=2) + P(X=3)$
  $= 0.15 + 0.05 = 0.2$
  $E(X) = \sum X * P(X) =$
  $(-2*0.1) + (-1*0.2) + (0*0.2) + (1*0.3) + (2*0.15) + (3*0.05)$

$= -0.2-0.2+0+0.3+0.3+0.15 = 0.35$

$E(X^2) = \sum X^{2} * P(X) =$

$(4*0.1) + (1*0.2) + (0*0.2) + (1*0.3) + (4*0.15) + (9*0.05)$

$= 0.4 + 0.2+0+0.3+0.6+0.45 = 1.95$

$V(x) = E(X^2) - [E(X)]^2$

$= 1.95 - (0.35)^2$

$= 1.8275$

Q 2. For the following distribution

Find P(X<1) , E(X) , V(X)

X	-1	0	1	2	3
P(X)	0.1	0.25	0.25	0.2	0.2

$P(X<1) = P(X= -1) + P(X=0) + P(X=1)$

$= 0.1 + 0.25 + 0.25 = 0.6$

$E(X) = \sum X * P(X) =$

$(-1*0.1) + (0*0.25) + (1*0.25) + (2*0.2) + (3*0.2)$

$= -0.1+0+0.25+0.4+0.6 = 1.15$

$E(X^2) = \sum X^{2} * P(X) =$

$(1*0.1) + (0*0.25) + (1*0.25) + (4*0.2) + (9*0.2)$

$= 0.1 + 0 + 0.25 + 0.8 + 1.8 = 2.95$

$V(x) = E(X^2) - [E(X)]^2$

$= 2.95 - (1.15)^2$

$= 1.6275$

Q 3. For the following probability distribution, find k , E(x)

X	0	1	2	3	4	5
P(X)	k	0.3	0.15	0.15	0.1	2k

Since P(X) is p.m.f , $\sum P(X) = 1$

$K + 0.3 + 0.15 + 0.15 + 0.1 + 2k = 1$

$3k + 0.7 = 1$

3k = 0.3

K = 0.1

We get p.m.f as

X	0	1	2	3	4	5
P(X)	0.1	0.3	0.15	0.15	0.1	0.2

$E(X) = \sum X * P(X) =$

$(0*0.1) + (1*0.3) + (2*0.15) + (3*0.15) + (4* 0.1) + (5 *0.2)$

$= 0+ 0.3+0.3+0.45+0.4+1.0 = 2.45$

Q4. Probability that Sheela wins Rs. 1000 in a competition is 0.6 and wins nothing is 0.4 , Find the Mathematical Expectation of her gain

X : Gain of Sheela

X	1000	0
P(X)	0.6	0.4

$E(X) = \sum X * P(X) =$

$(1000*0.6) + (0*0.4) = 600$

Q5 . A fast food center owner earns Rs. 10000 per day in festive season and Rs. 2000 per day for remaining part of the year. If probability of festive season is 0.4 , what is the Expected gain of the owner?

X : Gain of the fast food center owner

X	10000	2000
P(X)	0.4	0.6

$E(X) = \sum X * P(X) =$

$(10000*0.4) + (2000*0.6) = 4000 + 1200 = 5200$

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